Salem Ali

2.51

Case Study 1: Heated Cylinder

Collaborators: Joushua Padilla, Fawaaz Shaffeeullah

December 9th, 2019

Temperature Analysis of Heated Cylinder in Transient and Steady State

1. Introduction

Within this case study, a cylinder was analyzed for two different scenarios. In the first scenario, the cylinder is insulated and the transient behaviour of the cylinder is analyzed. In the second scenario, the cylinder is exposed to natural convection and radiation and the steady state temperature of the cylinder is predicted. Within both of these scenarios, a heat flux is applied at one side of the cylinder and the temperature is experimentally taken at different lengths away from this heat flux. In the first scenario, the linear time-variant lumped conductance model is used to analyze and predict the behaviour of the cylinder. The main purpose of the analysis is to predict the temperature in the cylinder at different times and positions. For the second scenario, Only the steady state behavior is being predicted and analyzed by modeling the cylinder as an isothermal cylinder with low radial and axial conductance, with the main modes of heat transfer being from the applied heat flux and dissipated via radiation and natural convection.

In terms of applications, there are a few cases in which this case is applicable. One clear one from the standpoint of a mechanical engineer is the use of a lathe. The heat flux in this applicable case is moreso a portion of the specific energy needed to remove material. While much of this specific energy is lost in the chip, a portion of the cutting energy dissipates into the stock itself, heating it up. Therefore, the representative heat flux can represent facing, turning, or drilling on one face of a cylinder. In a more general case, the cylinder is a body that can be used to represent a variety of common day items such as handles. In analysis, many bodies will be generalized to be a cylinder for the purpose of analysis given its very explored thermal interactions; this case study therefore provides a modeling of these generalized cylinders.

2. Experimental Setup

For this experiment, the setup and mounting of both scenarios was very similar. In both scenarios, an Aluminum 6061 cylinder is clamped between two rubber mounts. The rubber mounts are used so as to isolate the cylinder with regards to conduction. In the first scenario, a styrofoam pipe enclosure element is used to insulate the cylinder from natural convection and radiation. In the second scenario, this insulation element is removed. The other difference between the scenarios is the time of interest. In the insulated case, the transient behaviour is of interest while in the uninsulated case, the steady state behavior is of interest.

The aluminum cylinder had two holes drilled into it for the insertion thermocouples as well as a slot created for the insertion of a heating element. The heating element used is a 24-volt FDM 3-D printer heater. The heating element is press fitted into the slot that was created. The heat flux itself was created via the joule effect and a 30 volt rated power source. With regards to the measuring device, two type-K thermocouples were inserted into the holes at different positions of the cylinder, and were connected to a DAQ board. The data was collected via a Labview Virtual Instrument. The output of the Labview was the temperature at each thermocouple over time. In the transient case, the power source outputted 12 volts and a current of 0.792 Amps. Using joule effect and given that it has a coefficient of performance of 1, all the power being outputted by the power source is converted to heat. This relation is given by equation (1).

$$P \equiv I * V = Q$$

From the Joule effect, the heat transfer rate could be easily calculated. A model of the cylinder is shown in figure 2.1 and the insulated setup is shown in figure 2.2.

Figure 2.1

Figure 2.2

Figure 2.3 below represents the experimental data collected for the transient scenario. Both the temperature at the root (closer to the heater) and tip (farther from the heater) are represented. It is important to note that the heating period occurs during the linear section of the graph, while the flat line before and declining segment after correspond with the heater being turned off.

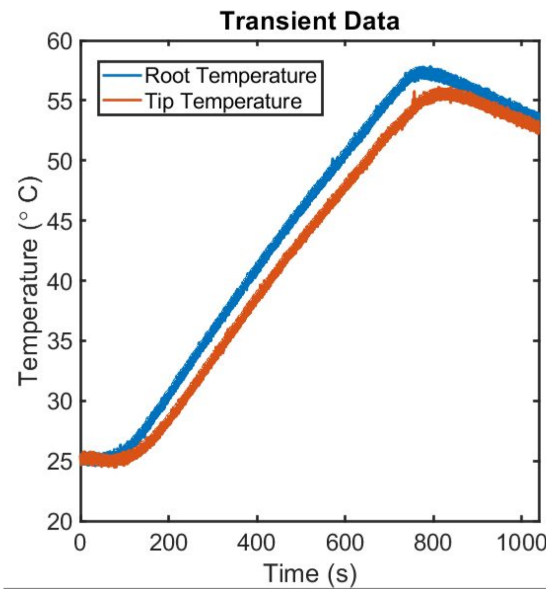

Figure 2.3

For the second scenario where the cylinder is not insulated, the foam insulation is removed. Otherwise, the setup remains the same and the temperature is measured at the time in which is stops changing over time. The heat generator was dissipating 5 volts with a current of 0.33 amps. The temperature measurement was taken at only one location given that at steady state and due to low a biot number in axial direction, there would be minimal temperature difference between the two thermocouples. Figure 2.4 represents the steady state temperature for scenario 2.

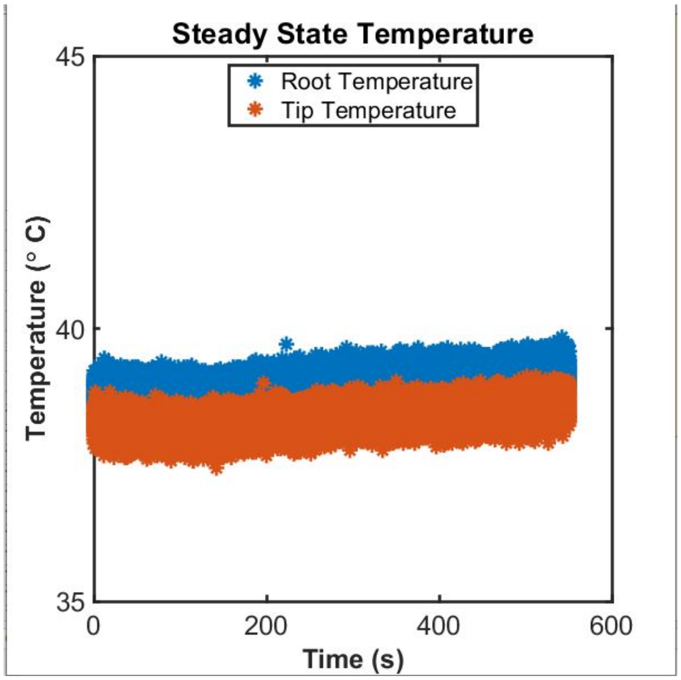

Figure 2.4

3. Analysis

Looking at the first scenario for the insulated cylinder, the problem is modeled using the lumped conductance case for insulated cylinders. The boundary conditions used for this problem is heat flux given that heat flux via the joule effect is the input condition within this experiment. The heat transfer rate can be found via equation (1) and is calculated below.

$$Q = I * V = 12 * 0.792 = 9.5 \ Watts$$

In order to find heat flux, the area over with to divide the heat transfer can either be the cross sectional area of the cylinder or the contact surface between the heater and the cylinder. Given that the cross sectional area of the cylinder is much easier to calculate, this option is used. Therefore, the heat flux is calculated below in equation (2).

$$q = Q/A_{cyl} = 4 * Q/(\pi * D_{cyl}^{2}) = 1.88 * 10^{4} Watts/m^{2} \ \textbf{(2)}$$

Given the problem class prescribed above, the temperature as a function of time is described by equations (3) and (4) below.

$$T(t) = T_{inf} + (T_{steady} - T_{inf}) * (1 - e^{-t/\tau}) \quad \textbf{(3)}$$

$$\tau = \rho * C * D_{cyl}/(4 * h) \ \textbf{(4)}$$

T_{steady} in this case represents the final steady state temperature. However, given that the cylinder has no meaningful method of removing heat as it is insulated, there is no steady state temperature. Therefore, the temperature function is altered to its applicable form in equation (5).

$$T_{avg}(t) = T_{inf} + (4Qt)/(\rho C \pi L D^2) \ \textbf{(5)}$$

T_{inf} is the temperature in the room, Q has been found, and all the other parameters are thermophysical properties. While this function is linear and suggests no upper bound, this is untrue at high temperatures. When the cylinder gets sufficiently hot, the heat will begin to dissipate through the insulation foam even given its extremely low thermal conductivity. In conjunction. In order to correctly use the lumped capacitance model, the model must be checked to ensure it is not acting like a semi-infinite model. Using equation (6), the time for which the cylinder acts like a semi-infinite body can be calculated.

$$t_{si} = .076 * L^2/\alpha = 4.5 \ seconds \ \textbf{(6)}$$

Given that data is collected over a much longer period than 4.5 seconds, the model is justified. Figure 3.1 represents the model temperature graphed alongside the experimental data.

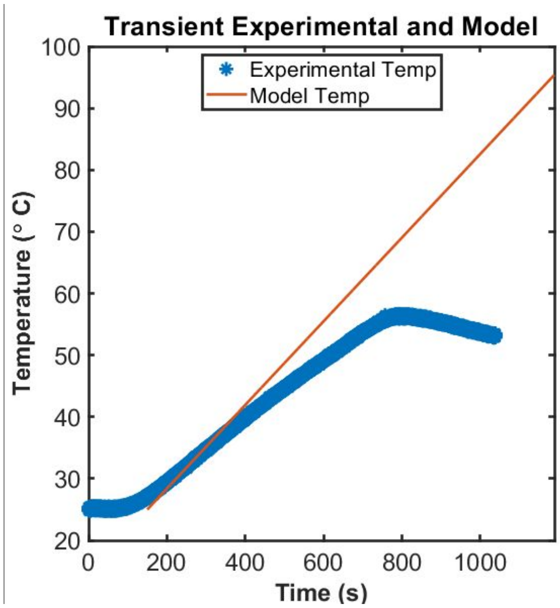

Figure 3.1

As can be seen, the model aligns with the experimental data very closely at first but begins to deviate heavily as the temperature increases. The most likely cause of this is due to the model assuming perfect insulation when in reality this is not the case. The insulation is a likely reason because the error increases proportionally with the temperature. Conduction, which is the form of heat transfer between the cylinder and foam, scales linearly with the temperature difference. Therefore, as the cylinder heats up, the temperature difference increases as well causing greater conductance into the foam. Overall, the foam not being a perfect insulation means the model overestimates the internal energy of the cylinder as it does not assume any heat dissipates, and thus overestimates the temperature.

With reference to the second scenario, the cylinder is now exposed to natural convection as well as radiation. Therefore, the cylinder will reach some steady state temperature where the heat transfer into the cylinder via the heater will be equal to the heat dissipated via convection and radiation. The problem class is assumed to be isothermal. The HTCs remain constant because this is within the steady state, so no temperatures are changing. While this is only true given the model passes certain conditions, these conditions are assumed to be true and will be checked later on. As mentioned above, the steady state temperature will occur when heat dissipated is equal to heat entering the cylinder. This is modeled in equation (7).

$$Q_{heater} = Q_{dissipated} = h_{total} * A_{surface} * (T_s - T_{inf}) \quad \textbf{(7)}$$

Solving for the heat transfer coefficient for natural convection first, the nusselt and rayleigh correlation are used to find the HTC. This process is shown in equations (8), (9), and (10). Given that both natural convections are functions of surface temperature, which is unknown, the HTCs be plugged into equation (7) so as to solve for the surface temperature.

$$Ra_D = g * \beta * D^3 * (T_s - T_{amb}) / (\nu * \alpha) \; \textbf{(8)}$$

$$Nu_D = 0.36 + 0.58 * Ra_D^{.25}(1 + (0.559/Pr)^{9/16})^{(-4/9)} \; \textbf{(9)}$$

$$h_{conv} = Nu_D * k_{air}/D = 1.11 * (2.263 * (T_s - 298.15)^{.25}) \quad \textbf{(10)}$$

Next, the radiation HTC must be found. Using existing correlations, equation (11) models the HTC of radiation well.

$$h_{rad} = 4\sigma\varepsilon * T_s^3 \; \textbf{(11)}$$

Now that both HTCs have been found as functions of the surface temperature, equation (7) can be used to solve for the surface temperature. Using equation (1), $Q = 5 * 0.33 = 1.65\ W$. Overall, $T_s = 315\ K$. This is graphed alongside the experimental data in figure 3.2.

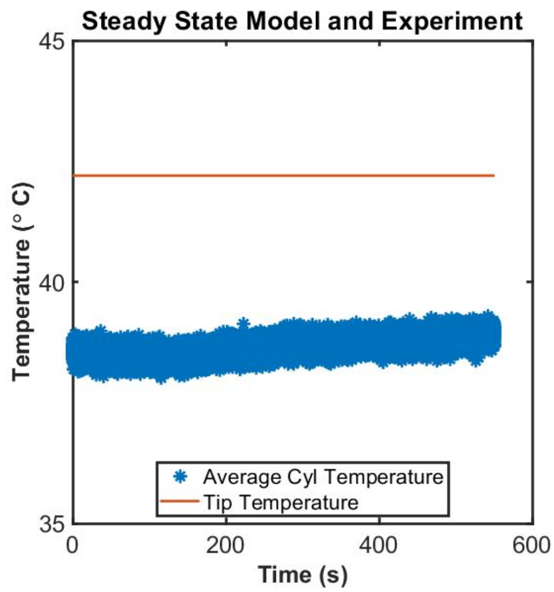

Figure 3.2

The experimental recorded a steady state temperature of around 312 K or 39 degrees C. Meanwhile, the model resulted in a steady state temperature of 315 K or 42 degrees C; therefore there was only an error of 3 degrees. The model was therefore quite accurate. Potential sources of error come from the calibration and thermal conductivity of the thermocouples, as they are not superconductors and thus don't represent the exact reading. Otherwise, the emissivity of the material may be underestimated in the model, leading to a higher steady state temperature.

As the final part of the analysis for the second scenario, the assumptions made for the isothermal cylinder must be checked. These assumptions were low biot in both the radial and axial directions as well as checking for an isothermal fin. These checks and their resultant calculations are shown in equations (12), (13), and (14). All three assumptions are true.

$$Bi_{rad} = h_{total}D/k_{Ll} << 1 \Rightarrow .00123 << 1 \quad \textbf{(12)}$$

$$Bi_{axial} = h_{total}L/k_{Al} << 1 \Rightarrow .0031 << 1 \quad \textbf{(13)}$$

$$3/M_{fin} >> L \Rightarrow 1.09 >> 0.11 \quad \textbf{(14)}$$

Properties of Air	**Value**
ν	$1.851E-5\ [m^2/s]$
α	$2.616E-5\ [m^2/s]$
Pr	0.71
K	$0.02821\ [W/mK]$
ρ	$1.07\ [kg/m^3]$
Properties of Aluminum	
K	$167\,[W/mK]$
ρ	$2781\,[kg/m^3]$
C	$883\ [J/kgK]$
ε	0.31
α	6.9E-5
Parameters	
T_{amb}	298.15 K
g	$9.8\ [m/s^2]$
β	1/295.15 [1/K]
σ	$5.667E-8$
L	4.5 in
D	1 in